6.2: COUNTING NAILS BY THE POUND

COUNTING BY WEIGHING AND AVOGADRO'S NUMBER

The size of molecule is so small that it is physically difficult, if not impossible, to directly count out molecules (Figure 6.2.1). However, we can count them indirectly by using a common trick of "counting by weighing".

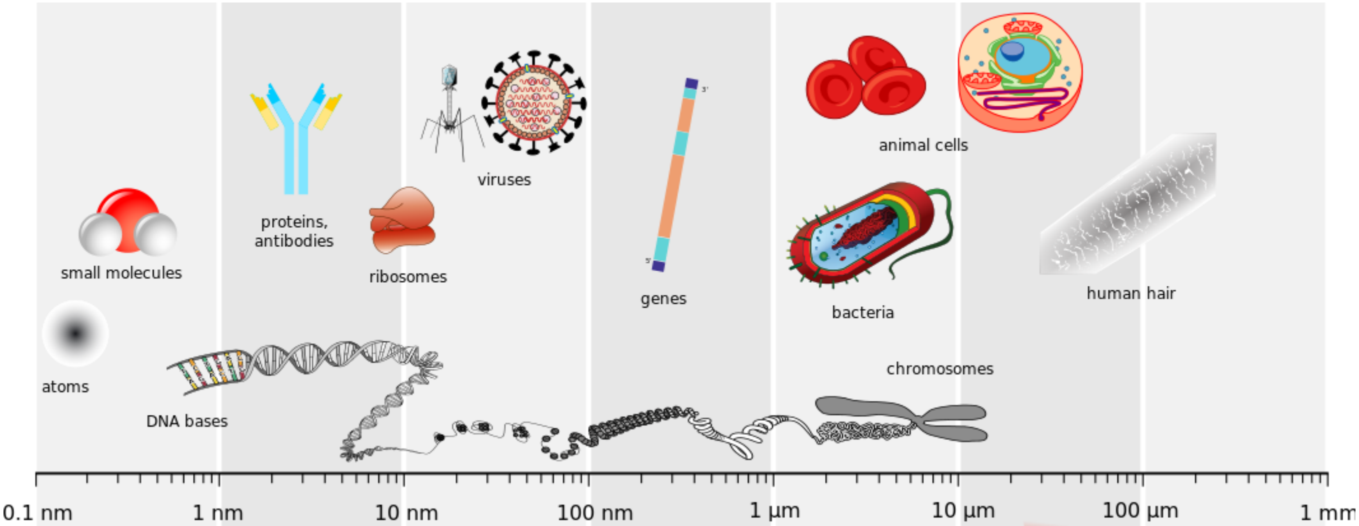

Figure 6.2.1: A comparison of the scales of various biological and technological objects. (CC BY-SA 3.0; Wikipedia)

Consider the example of counting nails in a big box at a hardware store. You need to estimate the number of nails in a box. The weight of an empty box is $213\,g$ and the weight of the box plus a bunch of big nails is $1340\,g$. Assume that we know that the weight of one big nail is 0.450 g. Hopefully it's not necessary to tear open the package and count the nails. We agree that

$$\text{mass of big nails} = 1340\,g - 213\,g = 1227\,g$$

Therefore

$$\text{Number of big nails in box} = \frac{1227\,g}{0.450\,g/\text{big nail}} = 2,726.6\,\text{big nails} = 2,730\,\text{big nails}. \tag{6.2.1}$$

You have just counted the number of big nails in the box by weighing them (rather than by counting them individually).

Figure 6.2.2: Galvanized nails. Individually counting nails in a box would would require significant effort. Alternatively, we can count them by weighing. (Public Domain; Wikipedia).

Now consider if the box of nails weighed the same, but the box were filled with small nails with an individual mass of $0.23\,g/\text{small nail}$ instead? You would do the same math, but use a different denominator in Equation 6.2.1:

$$\text{Number of small nails in box} = \frac{1227\,g}{0.230\,g/\text{small nail}} = 5,334.7\,\text{small nails} = 5,335\,\text{small nails}. \tag{6.2.2}$$

The individual mass is the conversion factor used in the calculation and changes, based on the nature of the nail (big or small). Let's ask a different question: how many *dozens* of nails are there in the same box of small nails described above?

If we know the information from Equation 6.2.2, we can just use the conversion of how many nails are in a dozen:

$$\frac{5,335 \text{ small nails}}{12 \text{ small nails/dozen}} = 444.6 \text{ dozen small nails} \tag{6.2.3}$$

If we want to get this value from weighing, we use the "dozen mass" instead of individual mass:

$$12 \times 0.23g = 2.76\, g/\text{dozen small nails}. \tag{6.2.4}$$

So following Equation 6.2.2, we get:

$$\text{Number of dozens of small nails} = \frac{1227\, g}{2.76\, g/\text{dozen small nails}} = 444.6 \text{ dozen small nails} \tag{6.2.5}$$

and this is the same result as Equation 6.2.3. These calculations demonstrate the difference between individual mass (i.e., per individual) and collective mass (e.g., per dozen or per gross). The collective mass of most importance to chemistry is *molar mass* (i.e., mass per mole or mass per 6.022×10^{23}).

AVOGADRO'S NUMBER

Avogadro's number is an accident of nature. It is the number of particles that delivers a mole of a substance. Avogadro's number = 6.022×10^{23} . The reason why the value is an accident of nature is that the mole is tied to the gram mass unit. The gram is a convenient mass unit because it matches human sizes. If we were a thousand times greater in size (like Paul Bunyan) we would find it handy to use kilogram amounts. This means the kilogram mole would be convenient. The number of particles handled in a kilogram mole is 1000 times greater. The kilo Avogadro number for the count of particles in a kilomole is 6.022×10^{26} .

If humans were tiny creatures (like Lilliputians) only 1/1000 our present size, milligrams would be more convenient. This means the milligram mole would be more useful. The number of particles handled in a milligram mole (millimole) would be 1/1000 times smaller. The milli Avogadro number for the count of particles in a millimole is 6.022×10^{20} .

What do you think would happen to Avogadro's number if the American system was used and amounts were measured in pound moles? Remember 1 pound = 454 grams. Avogadro's number would be larger by a factor of 454. A pound mole of hydrogen would weigh 1 pound, which is 454 grams. A gram mole of hydrogen weighs 1 gram and contains 6.022×10^{23} H atoms.

MOLAR MASS FOR ELEMENTS

You are able to read the periodic table and determine the average atomic mass for an element like carbon. The average mass is 12.01 amu. This mass is a ridiculously tiny number of grams. It is too small to handle normally. The molar mass of carbon is defined as the mass in grams that is numerically equal to the average atomic weight. This means

$$1g/mole carbon = 12.01\, g\, carbon$$

this is commonly written

$$1\, mol\ carbon = 12.01\, grams\ carbon.$$

This is the mass of carbon that contains 6.022×10^{23} carbon atoms.

- Avogadro's number is 6.022×10^{23} particles.

This same process gives us the molar mass of any element. For example:

- $1\, mol\ neon = 20.18\, g\, neon\, Ne$
- $1\, mol\ sodium = 22.99\, g\, sodium\, Na$

MOLAR MASS FOR COMPOUNDS

EXAMPLE 6.2.1: MOLAR MASS OF WATER

The formulas for compounds are familiar to you. You know the formula for water is H_2O. It should be reasonable that the weight of a formula unit can be calculated by adding up the weights for the atoms in the formula.

Solution

The formula weight for water

weight from hydrogen + weight from oxygen

The formula weight for water

2 H atoms x 1.008 amu + 1 O atom x 16.00 amu = 18.016 amu

The molar mass for water

18.016 grams water or 18 grams to the nearest gram

EXAMPLE 6.2.2: MOLAR MASS OF METHANE

The formula for methane, the major component in natural gas, is CH_4.

Solution

The formula weight for methane

weight from hydrogen + weight from carbon

The formula weight for methane

4 H atoms x 1.008 amu + 1 C atom x 12.01 amu = 16.04 amu

The molar mass for methane

16.04 grams per mole of methane

EXAMPLE 6.2.3: MOLAR MASS OF ETHYL CHLORIDE

What is its molar mass for ethyl chloride CH_3CH_2Cl?

Solution

The *formula weight*

weight from hydrogen + weight from carbon + weight from chlorine

The formula weight

5 H atoms x 1.008 amu + 2 C atom x 12.01 amu + 35.5 amu = 64.5 amu

The molar mass for ethyl chloride

64.5 grams per mole of ethyl chloride

EXAMPLE 6.2.4: MOLAR MASS OF SULFUR DIOXIDE

What is the molar mass for sulfur dioxide, $SO_2(g)$, a gas used in bleaching and disinfection processes?

Solution

Look up the atomic weight for each of the elements in the formula.

- 1 sulfur atom = 32.07 amu
- 1 oxygen atom = 16.00 amu

Count the atoms of each element in the formula unit.

- one sulfur atom
- two oxygen atoms

The formula weight

weight from sulfur + weight from oxygen

The formula weight

1 sulfur atom x (32. 07 amu) + 2 oxygen atoms x (16.00 amu)

The formula weight

SO_2 = 32. 07 amu + 32.00 amu = 64.07 amu = 64 amu SO_2

The molar mass for SO_2 is

$$64.07 \text{ grams of } SO_2; 1 \text{ mol } SO_2 = 64 \text{ grams per mole of } SO_2$$

? EXERCISE 6.2.1

What is the formula weight and molar mass for alum, $KAl(SO_4)_2 \bullet 12\,H_2O$?

Answer

1. Check the periodic table for the atomic masses for each atom in the formula.
2. Count the number of each type of atom in the formula.
3. Multiply the number of atoms by the atomic mass for each element.
4. Add up the masses for all of the elements.

Table 6.2.1: Masses of each element in alum, $KAl(SO_4)_2 \bullet 12\,H_2O$

element	average atomic mass	number of atoms in formula	rounded to nearest one unit for simplicity
potassium k	39.1 amu	1	39. amu
aluminum	26.98 amu	1	27. amu
sulfur	32.07 amu	2	64. amu
oxygen	16.00 amu	8 + 12 = 20	320. amu
hydrogen	1.008 amu	2 x 12 = 24	24. amu

Molar mass is 474 grams (add up the amu of each element to find the total of 474 amu). This is a mass in grams that is numerically (474) the same as the formula weight.

1 mole alum $KAl(SO_4)_2 \bullet 12\,H_2O$ = 474 grams alum $KAl(SO_4)_2 \bullet 12\,H_2O$